



University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 1 EXAMINATION 2015/2016

MATH00030

Access to Science, Engineering and Agriculture: Mathematics 1

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Time Allowed: 2 hours

Instructions for Candidates

Candidates should attempt all questions.

Note that not all questions are allocated the same number of marks.

Notes for Invigilators

Non programmable calculators are permitted.

The formula sheet provided is permitted.

1. (a) Without using a calculator, calculate the following.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator.

(i) $\frac{2}{9} - \frac{4}{5}$

(ii) $\frac{7}{5} \times \left(-\frac{6}{11}\right)$

(iii) $\frac{4}{5} \div \frac{5}{4}$

(iv) -3^2

(v) $\left(\frac{9}{4}\right)^{-\frac{3}{2}}$

(vi) $7 + 7 \div (-6) + 3$

(vii) $\log_3 27$

(viii) $\log_2 \frac{1}{4}$ [8]

- (b) Simplify the following expressions by expressing them as a power of x and/or y , as appropriate.

(i) $x^3 \times x^{-1}$

(ii) $x^{\frac{1}{2}} \div x^{-\frac{1}{3}}$

(iii) $(x^2)^{-3}$

(iv) $(\sqrt[3]{xy})^3$ [5]

- (c) (i) Approximate 15.9950 to two decimal places.

- (ii) Approximate 0.0002345 to two significant figures.

- (iii) Express 1530.13 in scientific notation.

- (iv) Express 0.0000205 in scientific notation to one significant figure. [4]

(d) Simplify $(x^2 - x - 1) - (-x + 2)$. [1]

(e) Multiply out $(x^4 + 3x^2)(-x + 2)$. [2]

(f) Perform long division on $\frac{x^2 + x + 1}{x + 2}$, giving the quotient and remainder. [4]

(g) Evaluate $\sum_{i=-1}^2 i^3$. [2]

(h) Calculate $\binom{8}{3}$ without using a calculator.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator. [2]

(i) Expand $(2x - y)^3$ using The Binomial Theorem. [4]

2. (a) Find the equation of the line through the point $(1, -3)$ parallel to the line $y = -2x - 1$. [2]

- (b) Solve the simultaneous equations

$$4x - y = 6$$

$$3x + 4y = -5$$

[3]

- (c) Find the length of the line segment between $(-2, 1)$ and $(-1, 2)$. [1]

3. (a) Write the expression $3x^2 - 2x - 5$ in completed square form. [3]

- (b) Solve the equation $3x^2 - 2x - 5 = 0$ by using the quadratic formula. [2]

- (c) Sketch the graph of the function $y = 3x^2 - 2x - 5$, showing the y -intercept, the x -intercept(s) (if applicable) and the turning point. [4]

4. (a) For each of the following:

- Say whether or not it is a function and if not say why not.
- If it is a function state the domain and the codomain.

(i)

$$f: \mathbb{R}^- \rightarrow \mathbb{R}^+$$

$$x \mapsto -x + 1$$

(ii)

$$f: \mathbb{R}^- \rightarrow \mathbb{R}^+$$

$$x \mapsto x^2 - 1$$

[4]

- (b) Sketch the graph of the function

$$f: \{-2, -1, 0, 1, 3\} \rightarrow \{1, 2, 4\}$$

$$-2 \mapsto 1$$

$$-1 \mapsto 4$$

$$0 \mapsto 1$$

$$1 \mapsto 4$$

$$3 \mapsto 1$$

[2]

(c) Figure 1 contains the graphs of four of the following functions:

(i) $y = 5^x$

(ii) $y = -6^x$

(iii) $y = \left(\frac{4}{7}\right)^x$

(iv) $y = -\left(\frac{2}{3}\right)^x$

(v) $y = \log_4(x)$

(vi) $y = \log_{1/5}(x)$

Match the functions to the graphs.

[4]

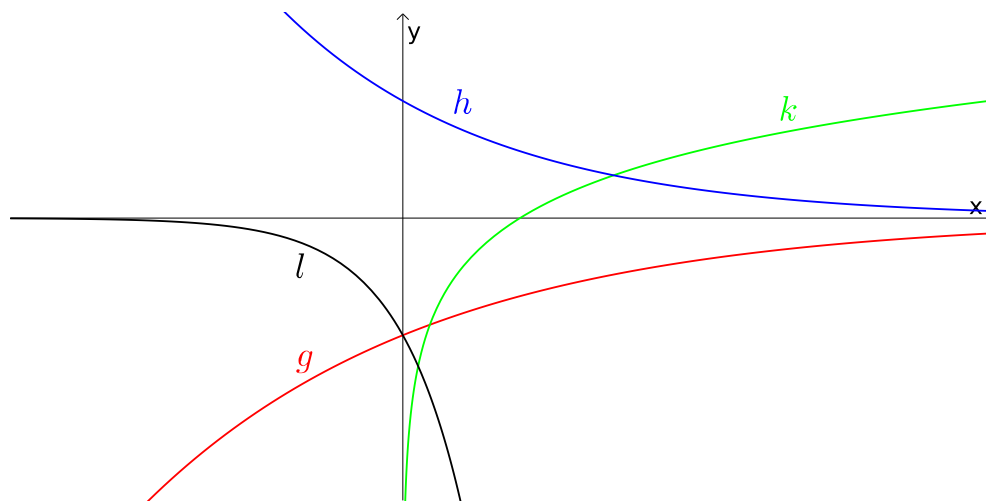


Figure 1: The functions for Question 4 (c).

(d) For each of the following functions, say whether they are injective, surjective or bijective. If a function is not injective or surjective then say why not.

(i)

$$f: \{A, B, C, D\} \rightarrow \{1, 2, 3, 4\}$$

$$A \mapsto 1$$

$$B \mapsto 4$$

$$C \mapsto 3$$

$$D \mapsto 1$$

(ii)

$$f: \mathbb{R}^- \rightarrow \mathbb{R}^-$$

$$x \mapsto x - 1$$

[3]

- (e) State whether each of the functions in Part (d) has an inverse function or not, giving a reason in each case. [1]
5. (a) Convert 300° to radians, leaving your answer as a multiple of π . [1]
- (b) Convert $\frac{7\pi}{12}$ radians to degrees. [1]
- (c) Using the geometric method, find $\cos\left(\frac{7\pi}{6}\right)$ without using a calculator. [4]
- (d) Using whichever trigonometric formulae you like, but without using a calculator, calculate the following.
Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator.
- (i) $\cos\left(\frac{5\pi}{6}\right)$
- (ii) $\sin\left(\frac{7\pi}{4}\right)$ [4]
- (e) Find the length of the side a in the triangle in Figure 2. [3]

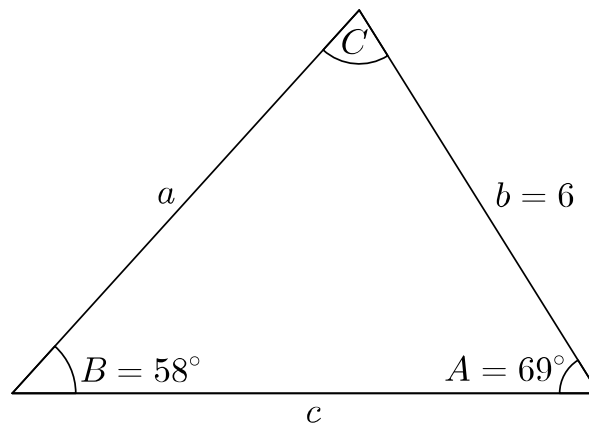


Figure 2: The triangle for Question 5 (e).

6. (a) Find the derivative of $f(x) = x^2$ using first principles. [3]
- (b) Find the derivatives of the following functions.
- (i) $f(x) = \ln 3$
- (ii) $f(x) = x^3$
- (iii) $f(x) = \ln(-2x)$ (where $x < 0$)
- (iv) $f(x) = \cos(-2x)$
- (v) $f(x) = 2x^{\frac{1}{2}} - \sin(-x) + 2e^{-4x}$ [6]

7. Find the following integrals.

(a) $\int 2 \, dx$ [1]

(b) $\int_1^2 x^5 \, dx$ [2]

(c) $\int_0^\pi \cos\left(\frac{1}{2}x\right) \, dx$ [2]

(d) $\int e^{2x} + x^{-\frac{3}{4}} \, dx$ [2]

8. (a) For the list of numbers 3, 2, -8, 8, 2, 5, -5, find the

(i) Mean

(ii) Median

(iii) Mode(s)

(iv) Interquartile range [5]

(b) Find the line of best fit using the least squares method with the points

$(-4, -2), (-2, -2), (0, 0), (3, 1)$ and $(4, 2)$. [8]

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